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Kaluza-Klein Holographic Dark Energy Cosmological models in Modified Theory of Gravity

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Abstract: *The main motive of this study is to investigate Kaluza-Klein metric within the presence of Holographic Dark Energy cosmological models in the framework of $f(R,T)$ theory of gravity. The exact solution of the field equations have been obtained by assuming two different volumetric expansion laws in a way to cover all possible expansion: namely exponential and power law expansion. Keeping an eye on the accelerating nature of the universe in the present epoch, the dynamics and physical behavior of the models have been discussed.*

Keywords: *Holographic Dark energy, f(R,T) gravity, Kaluza-Klein.*

I. Introduction

Red shift supernova Ia [1-7], Cosmic Microwave Background Radiation [8-9] and Large Scale Structure [10-15] have shown that our universe is currently accelerating. After estimating various energy components of the Universe, the cause of its accelerated expansion has been attributed to some exotic energy stuff dubbed dark energy (DE). DE yields an isotropic pressure and obeys a simple EoS in the form p = wρ, where ρ is the energy density, p is the isotropic pressure and w is the EoS parameter, which is not necessarily constant. Several modified theories of gravity have been developed and studied, in view of the late time acceleration of the Universe and the existence of dark energy and dark matter.

Modify gravity is of great importance because it can successfully explain the rotation curve of galaxies and the motion of galaxy clusters in the universe. There are various modify gravity namely $f(R)$, $f(G)$, $f(R,G)$, $f(T)$ and $f(R,T)$ theory of gravity. Harko et al. [16] developed a $f(R,T)$ modified theory of gravity, where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and of the trace T of the stress energy tensor. The $f(R,T)$ gravity model depends on a supply term, representing the variation of the matter stress energy tensor with regard to the metric. A general expression for this supply term is obtained as an operate of the matter Lagrangian L_m in order that every selection of L_m would generate a particular set of field equations. Point like Lagrangian's for $f(R,T)$ gravity had been presented by Myrzakulov [17]. The $f(R,T)$ gravity model that satisfies the local tests and transition of matter from dominated era to accelerated phase was considered by Houndjo [18]. Adhav [19] has obtained LRS Bianchi type I cosmological model in $f(R,T)$ gravity. Bianchi type III cosmological model in $f(R,T)$ gravity have been discussed by Reddy et al. [20]. Many physicists [21-53] have investigated $f(R,T)$ gravity in different contexts.

In recent years, holographic dark energy (HDE) models have received considerable attention to describe dark energy cosmological models. Several aspects of holographic dark energy have been investigated by Cohen et al. [54] ,Hsu [55], Gao et al. [56].Granda and Olivers [57] proposed a holographic density of the form $\rho_{DE} \approx \left(\alpha_1 H^2 + \beta_1 \dot{H}\right)$ where H is the Hubble parameter and α_1, β_1 are constants which must satisfy the restrictions imposed by the current observational data. Several relativists [58-70] studied various aspects of Holographic Dark Energy (HDE) cosmological models in general relativity and scalar tenor theory of gravitation.

Inspiring by above investigations, we consider higher dimensional Kaluza-Klein holographic dark energy model $f(R,T)$ theory of gravitation. In section 2, we present gravitational $f(R,T)$ field equations. In section 3, we obtain the field equations of Kaluza-Klein metric. The solution of the field equations are dealt in section 4.Power law and exponential law are studied in section 5 and 6 respectively. Section 7 is referred to the findings of the power and exponential models. Conclusions of the obtained models are presented in section 8.

2. Gravitational field equations of $f(R,T)$ gravity

The $f(R,T)$ gravity is the generalization of General Relativity (GR). In this theory, the field equations are derived from a variation, Hilbert-Einstein type principle which is given as

$$S = \frac{1}{16\pi}\int \sqrt{-g}\, f(R,T)\, d^4x + \int \sqrt{-g}\, L_m d^4x , \tag{1}$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar (R) and trace of the stress energy tensor (T) of the matter T_{ij} ($T = g^{ij}T_{ij}$) and L_m is the matter Lagrangian density.

The stress energy tensor of matter is defined as

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\,L_m)}{\delta g^{ij}} . \tag{2}$$

Assuming that the Lagrangian density L_m of matter depends only on the metric tensor components g_{ij} and not on its derivatives, in this case we obtain

$$T_{ij} = g_{ij}L_m - \frac{\delta(L_m)}{\delta g^{ij}} . \tag{3}$$

The $f(R,T)$ gravity field equations are obtained by varying the action S with respect to the metric tensor components g_{ij} ,

$$f_R(R,T)R_{ij} - \frac{1}{2} f(R,T)g_{ij} + f_R(R,T)\left(g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j\right) = 8\pi T_{ij} - f_T(R,T)T_{ij} - f_T(R,T)\Theta_{ij} , \tag{4}$$

where

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{\alpha\beta}\frac{\partial^2 L_m}{\delta g^{ij}\partial g^{\alpha\beta}} . \tag{5}$$

Here $f_R = \frac{\delta f(R,T)}{\delta R}$, $f_T = \frac{\delta f(R,T)}{\delta T}$ $\Theta_{ij} = g^{\alpha\beta}\frac{\delta T_{\alpha\beta}}{\delta g^{ij}}$ and ∇_i is the covariant derivative.

The contraction of equation (4) yields

$$f_R(R,T)R + 3\Pi f_R(R,T) - 2f(R,T) = (8\pi - f_T(R,T))T - f_T(R,T)\Theta \text{ with } \Theta = g^{ij}\Theta_{ij} . \tag{6}$$

Equation (6) gives a relation between Ricci scalar and the trace of energy momentum tensor.

Using matter Lagrangian L_m the stress energy tensor of the matter is given by

$$T_{ij} = (p+\rho)u_i u_j - pg_{ij} , \tag{7}$$

where $u^i = (0,0,0,1)$ denotes the four velocity vector in co-moving coordinates which satisfies the condition $u^i u_i = 1$. ρ and p is energy density and pressure of the fluid respectively.

The variation of stress energy of perfect fluid has the following expression

$$\Theta_{ij} = -2T_{ij} - pg_{ij} . \tag{8}$$

On the physical nature of the matter field, the field equations also depend through the tensor Θ_{ij} . Several theoretical models corresponding to different matter contributions for $f(R,T)$ gravity are possible. However, Harko et al. [16] gave three classes of these models

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)\,f_3(T) \end{cases} . \tag{9}$$

In this paper, we have focused to the first class $f(R,T)=R+2f(T)$, where $f(T)$ is an arbitrary function of tress energy tensor of the form $f(T)=\mu T$ where μ is constant. For this choice the gravitational field equations of $f(R,T)$ gravity becomes

$$R_{ij}-\frac{1}{2}Rg_{ij}=8\pi T_{ij}-2\dot{f}(T)T_{ij}-2\dot{f}(T)\Theta_{ij}+f(T)g_{ij}, \tag{10}$$

where the dot denotes differentiation with respect to the argument. If the matter source is a perfect fluid then the field equations (in view of Eq. (8)) becomes

$$R_{ij}-\frac{1}{2}Rg_{ij}=8\pi T_{ij}+2f'(T)T_{ij}+[2pf'(T)+f(T)]g_{ij}. \tag{11}$$

3. Field equations of Kaluza- Klein metric

We consider five dimensional Kaluza-Klein metric

$$ds^2=dt^2-A^2\left(dx^2+dy^2+dz^2\right)-B^2d\psi^2, \tag{12}$$

where A and B are functions of t only.

We choose the function $f(T)$ of the trace of the stress–energy tensor of the matter so that

$$f(T)=\mu T \tag{13}$$

where μ is a constant (Harko et. al.[16]).

For matter and holographic dark energy, the energy momentum tensors are defined as $T_{\mu\nu}=\rho_m u_\mu u_\nu$; $\overline{T}_{\mu\nu}=(\rho_\Lambda+p_\Lambda)u_\mu u_\nu+g_{\mu\nu}p_\Lambda$, (14)

where ρ_m is the energy densities of matter, ρ_Λ is the holographic dark energy and p_Λ is the pressure of the holographic dark energy.

The holographic dark energy density of the form

$$\rho_\Lambda=3(\alpha H^2+\beta\dot{H}), \tag{15}$$

where H is the Hubble parameter and α,β are constants.

The continuity equations can be obtained as

$$\dot{\rho}_m+\dot{\rho}_\Lambda+3H(\rho_m+\rho_\Lambda+p_\Lambda)=0, \tag{16}$$

$$\dot{\rho}_\Lambda+3H(\rho_\Lambda+p_\Lambda)=0. \tag{17}$$

The barotropic equation of state is

$$p_\Lambda=\omega_\Lambda\rho_\Lambda. \tag{18}$$

From equations (15)-(18), the EoS parameter become

$$\omega_\Lambda=-1-\frac{2\alpha H\dot{H}+\beta\ddot{H}}{3H(\alpha H^2+\beta\dot{H})}. \tag{19}$$

Holographic dark energy principal should be restricted by an infrared cutoff scale L and ultraviolet cutoff scale Λ without decaying into a black hole. The quantum vacuum energy should be less than or equal to the mass of a black hole that is $L^3\rho_\Lambda\le LM_p^2$, where ρ_Λ is the vacuum energy density and $M_p=(8\pi G)^{-1/2}$ is the reduced plank mass.

Using commoving coordinates and equations (14)–(15) and (13), the $f(R,T)$ gravity field equations, (11), for metric (12) can be written as

$$2\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+2\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{A}^2}{A^2}=8\pi p_\Lambda-[-4p_\Lambda+\rho_m+\rho_\Lambda]\mu \tag{20}$$

$$3\frac{\ddot{A}}{A}+3\left(\frac{\dot{A}}{A}\right)^2=8\pi p_\Lambda-[-4p_\Lambda+\rho_m+\rho_\Lambda]\mu \tag{21}$$

$$3\left(\frac{\dot{A}}{A}\right)^2+3\frac{\dot{A}}{A}\frac{\dot{B}}{B}=-8\pi(\rho_m+\rho_\Lambda)-[-2p_\Lambda+3\rho_m+3\rho_\Lambda]\mu \tag{22}$$

where a dot here in after denotes ordinary differentiation with respect to cosmic time "t" only.

4. Isotropization and the solution

The isotropy of the expansion can be parametrized after defining the directional Hubble's parameters and the average Hubble's parameter of the expansion. The directional Hubble parameters in the directions x, y, z, ψ for the Kaluza-Klein metric defined in (12) may be defined as follows:

$$H_x = H_y = H_z = \frac{\dot{A}}{A} \text{ and } H_\psi = \frac{\dot{B}}{B} \tag{23}$$

The mean Hubble parameter, H, is given by

$$H = \frac{\dot{R}}{R} = \frac{1}{4}\frac{\dot{V}}{V} = \frac{1}{4}\left(3\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) \tag{24}$$

where R is the mean scale factor and $V = R^4 = A^3 B$ is the spatial volume of the universe.

The anisotropy parameter of the expansion Δ is defined as

$$\Delta = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{H_i - H}{H}\right)^2 \tag{25}$$

in the x, y, z, ψ directions, respectively. The mean anisotropic parameter of the expansion Δ has a very crucial role in deciding whether the model is isotropic or anisotropic. It is the measure of the deviation from isotropic expansion, the universe expands isotropically when $\Delta = 0$.

Let us introduce the dynamical scalars, such as expansion parameter (θ) and the shear (σ^2) as usual

$$\theta = 4H \tag{26}$$

$$\sigma^2 = 2\Delta H^2 \tag{27}$$

Equations (20) and (21) lead to

$$\frac{\ddot{A}}{A} + \frac{2\dot{A}^2}{A^2} - \frac{2\dot{A}\dot{B}}{AB} - \frac{\ddot{B}}{B} = 0,$$

$$\frac{d}{dt}\left[\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right] = -\left(\frac{A}{A} - \frac{\dot{B}}{B}\right)\left[\frac{3\dot{A}}{A} + \frac{\dot{B}}{B}\right]. \tag{28}$$

Let V be the function of t defined by

$$V = A^3 B\ . \tag{29}$$

Then from equation (29), we obtain

$$\frac{d}{dt}\left[\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right] = -\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\frac{\dot{V}}{V}. \tag{30}$$

Integrating the above equation, we get

$$\frac{A}{B} = d\exp\left[x\int\frac{1}{V}dt\right], \tag{31}$$

where x and d are constants of integration.

In view of $V = A^3 B$, equation (31) leads to

$$A = d^{\frac{1}{4}} V^{\frac{1}{4}} \exp\left[\frac{x}{4}\int\frac{dt}{V}\right], \tag{32}$$

$$B = d^{\frac{-3}{4}} V^{\frac{1}{4}} \exp\left[\frac{-3}{4}x\int\frac{dt}{V}\right]. \tag{33}$$

Using equations (32) and (33), A and B are explicitly be expressed as

$$A = d_1 V^{\frac{1}{4}} \exp\left[X_1 \int \frac{dt}{V}\right], \tag{34}$$

$$B = d_2 V^{\frac{1}{4}} \exp\left[X_2 \int \frac{dt}{V}\right], \tag{35}$$

where

$$d_1{}^3 d_2 = 1, \quad 3X_1 + X_2 = 0, \quad d_1 = d^{\frac{1}{4}}, \quad d_2 = d^{-\frac{3}{4}}, \quad X_1 = \frac{x}{4}, X_2 = \frac{-3x}{4}.$$

The above equations become

$$A = D_1 V^{\frac{1}{4}} \exp\left[X_1 \int \frac{dt}{V}\right], \tag{36}$$

$$B = D_2 V^{\frac{1}{4}} \exp\left[X_2 \int \frac{dt}{V}\right], \tag{37}$$

where $D_1^3 D_2 = 1$ and $3X_1 + X_2 = 0$.

Since the field equations (20)–(22) are three equations having four unknowns and are highly nonlinear, an extra condition is needed to solve the system completely. Here we have used two different volumetric expansion laws

$$V = at^b \tag{38}$$

and

$$V = \alpha_1 e^{\beta_1 t}, \tag{39}$$

where a, b, α_1, β_1 are constants. In this way, all possible expansion histories, the power law expansion, (28), and the exponential expansion, (39), have been covered.

5. Model for power law

Using (38) in (36) and (37), we obtain the scale factors as follows:

$$A = D_1 \left[a^{\frac{1}{4}} t^{\frac{b}{4}}\right] \exp\left\{\frac{X_1}{a(1-b)} t^{1-b}\right\} \tag{40}$$

and

$$B = D_2 \left[a^{\frac{1}{4}} t^{\frac{b}{4}}\right] \exp\left\{\frac{X_2}{a(1-b)} t^{1-b}\right\}. \tag{41}$$

Metric (12) with the help of (40) and (41) can be written as

$$ds^2 = dt^2 - D_1^2 a^{2/4} t^{2b/4} \exp\left\{2\left[\frac{X_1}{a(1-b)} t^{1-b}\right]\right\}\left(dx^2 + dy^2 + dz^2\right) - D_2^2 a^{2/4} t^{2b/4} \exp\left\{2\left[\frac{X_2}{a(1-b)} t^{1-b}\right]\right\} d\psi^2. \tag{42}$$

The directional Hubble parameters are found as

$$H_x = H_y = H_z = \frac{b}{4t} + \frac{X_1}{at^b} \tag{43}$$

$$H_\psi = \frac{b}{4t} + \frac{X_2}{at^b} \tag{44}$$

The mean Hubble's parameter, H, is given by

$$H = \frac{b}{4t}. \tag{45}$$

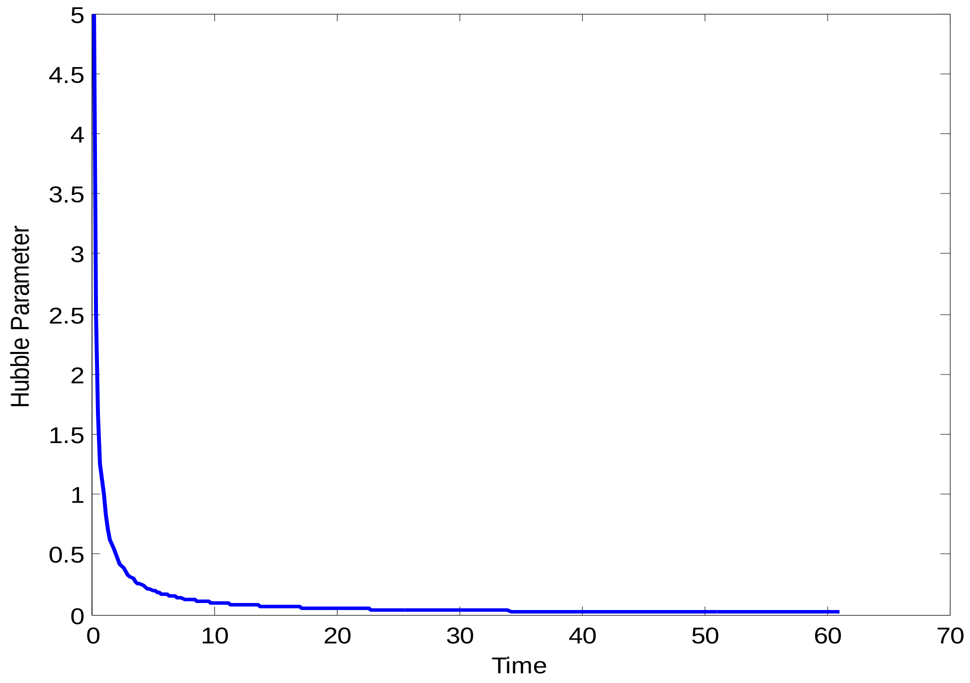

Figure No. 1. Hubble parameter vs Time.

Using the directional and mean Hubble's parameter in (25), we obtain

$$\Delta = \frac{4X^{2}}{a^{2}b^{2}t^{2(b-1)}}. \tag{46}$$

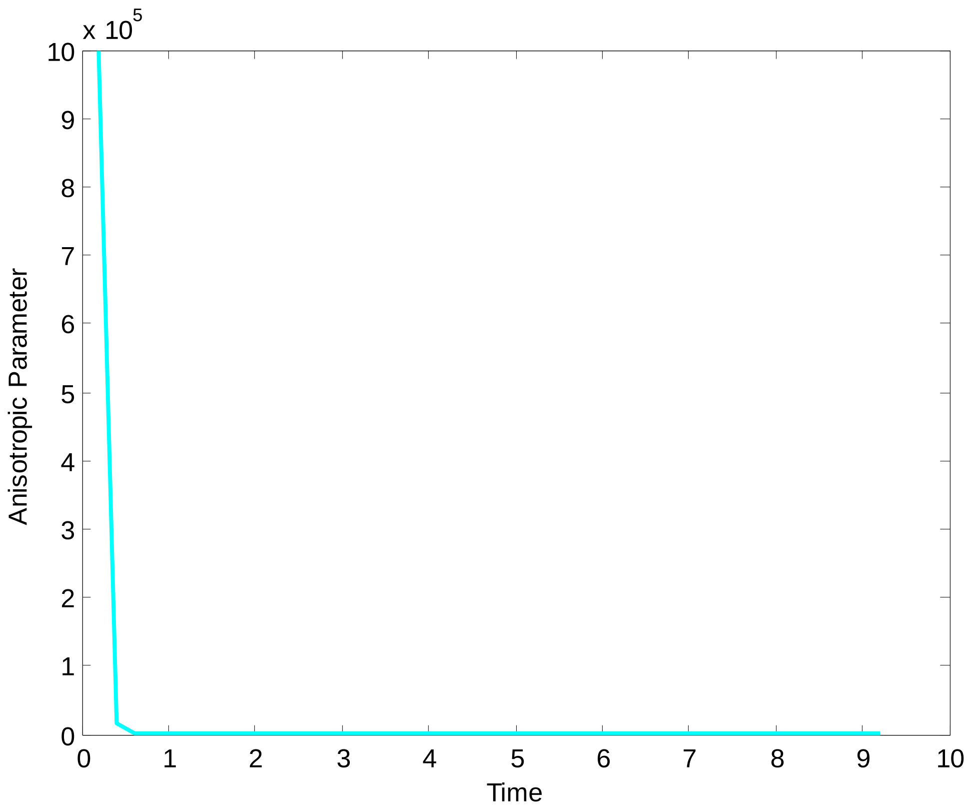

Figure No. 2. Anisotropic parameter vs Time.

The dynamical scalars are given by

$$\theta = \frac{b}{t}. \tag{47}$$

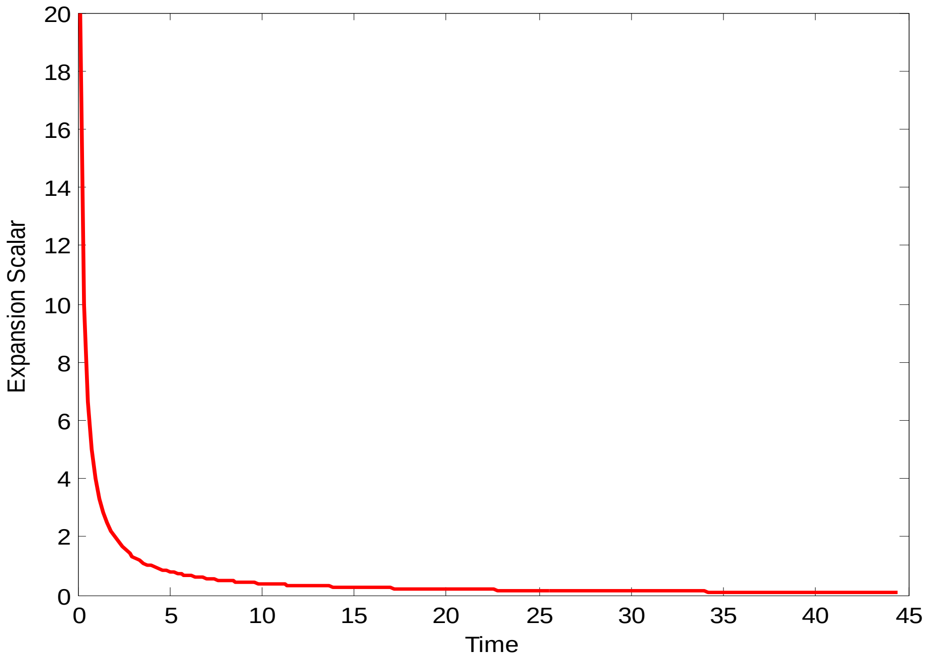

Figure No. 3. Expansion Scalar vs Time.

$$\sigma^2 = \frac{X^2}{2a^2t^{2b}}, \tag{48}$$

where $X^2 = 3X_1^2 + X_2^2 = \text{constant}$.

The deceleration parameter

$$q = \frac{4}{b} - 1, \tag{49}$$

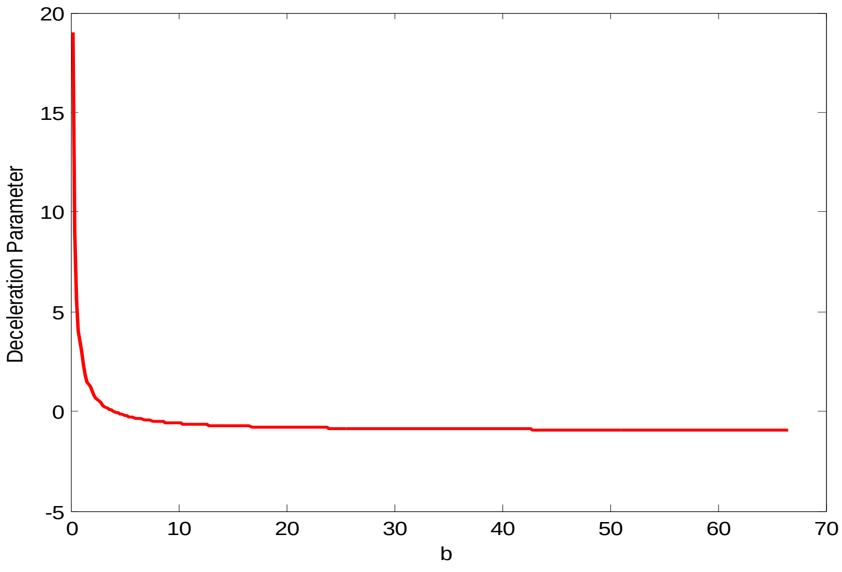

Figure No. 4. Deceleration Parameter vs b.

The holographic dark energy density and pressure become

$$\rho_\Lambda = \frac{3b(\alpha b - 4\beta)}{16t^2}, \quad (50)$$

$$p_\Lambda = \left(-1 + \frac{8}{3b}\right)\left(\frac{3b(\alpha b - 4\beta)}{16t^2}\right). \quad (51)$$

The EoS parameter yields

$$\omega_\Lambda = -1 + \frac{8}{3b} \quad (52)$$

We obtain the energy density of matter as

$$\rho_m = \frac{k_1}{at^b} \quad (53)$$

The matter density parameter Ω_m and holographic dark energy density parameter Ω_Λ are given by

$$\Omega_m = \frac{\rho_m}{4H^2} = \frac{4k_1}{ab^2} t^{2-b}, \quad (54)$$

$$\Omega_\Lambda = \frac{\rho_\Lambda}{4H^2} = \frac{3(\alpha b - 4\beta)}{4b}. \quad (55)$$

From equations (54) and (55), we get overall density parameter

$$\Omega = \Omega_m + \Omega_\Lambda = \frac{4k_1}{ab^2} t^{2-b} + \frac{3(\alpha b - 4\beta)}{4b}. \quad (56)$$

6. Model for exponential law

Using (39) in (36) and (37), we obtain the scale factors as follows:

$$A = D_1\left[\alpha_1^{\frac{1}{4}} e^{\beta_1 \frac{t}{4}}\right] \exp\left\{\frac{-X_1}{\alpha_1\beta_1} e^{-\beta_1 t}\right\} \quad (57)$$

and

$$B = D_2\left[\alpha_1^{\frac{1}{4}} e^{\beta_1 \frac{t}{4}}\right] \exp\left\{\frac{-X_2}{\alpha_1\beta_1} e^{-\beta_1 t}\right\}. \quad (58)$$

Metric (12) with the help of (57) and (58) can be written as

$$ds^2 = dt^2 - D_1^2 \alpha_1^{\frac{1}{2}} e^{\frac{\beta_1 t}{2}} \exp\left\{-2\left[\frac{X_1}{\alpha_1\beta_1} e^{-\beta_1 t}\right]\right\}\left(dx^2 + dy^2 + dz^2\right) - D_2^2 \alpha_1^{\frac{1}{2}} e^{\frac{\beta_1 t}{2}} \exp\left\{-2\left[\frac{X_2}{\alpha_1\beta_1} e^{-\beta_1 t}\right]\right\} d\psi^2. \quad (59)$$

The directional Hubble parameters are found as

$$H_x = H_y = H_z = \frac{\beta}{4} + \frac{X_1}{\alpha_1 e^{\beta_1 t}} \quad (60)$$

$$H_\psi = \frac{\beta}{4} + \frac{X_2}{\alpha_1 e^{\beta_1 t}} \quad (61)$$

The mean Hubble's parameter, H, is given by

$$H = \frac{\beta}{4}. \quad (62)$$

The anisotropy parameter of the expansion, Δ, is

$$\Delta = \frac{4X^2 e^{-2\beta_1 t}}{\alpha_1^2 \beta_1^2}. \quad (63)$$

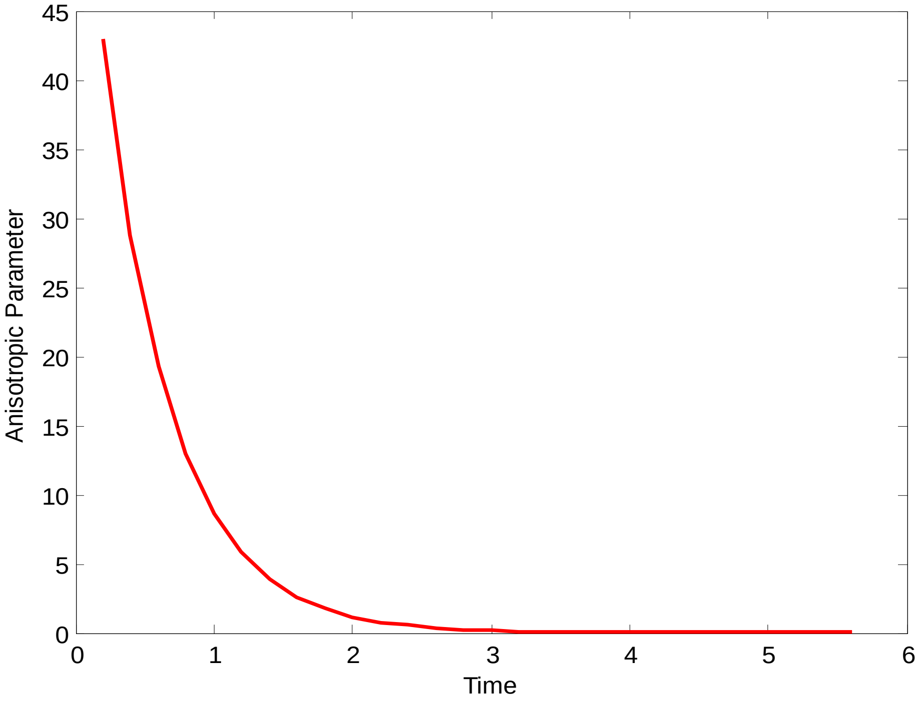

Figure No. 5. Anisotropic Parameter vs Time.

The expansion scalar, θ , is found as

$$\theta = \beta \ . \tag{64}$$

The shear scalar, σ^2 , is obtained as

$$\sigma^2 = \frac{X^2 e^{-2\beta_1 t}}{2\alpha_1^2} . \tag{65}$$

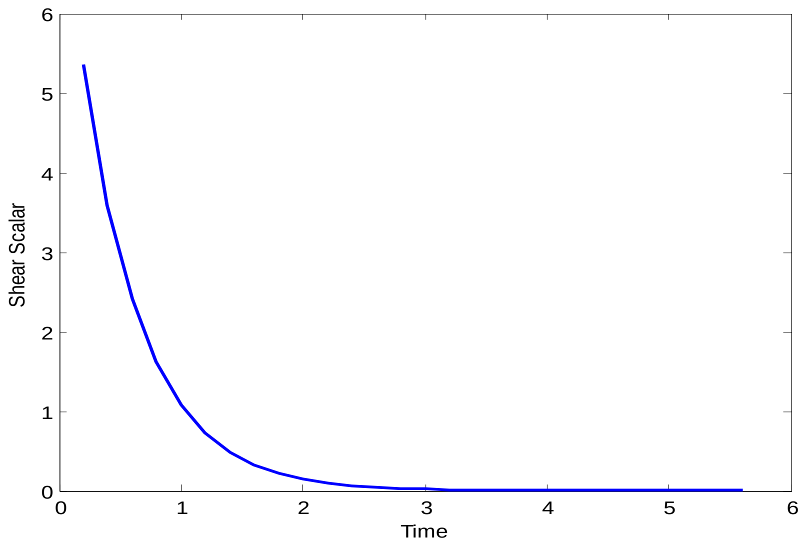

Figure No. 6. Shear Scalar vs Time.

The deceleration parameter

$$q = -1 \ , \tag{66}$$

where $X^2 = 3X_1^2 + X_2^2$ = constant .

The holographic dark energy density and pressure become

$$\rho_\Lambda = \frac{3\alpha\beta^2}{16}, \tag{67}$$

$$p_\Lambda = -\frac{3\alpha\beta^2}{16}. \tag{68}$$

The EoS parameter yields

$$\omega_\Lambda = -1. \tag{69}$$

We obtain the energy density of matter as

$$\rho_m = \frac{k_2}{\alpha_1 e^{\beta_1 t}} \tag{70}$$

The matter density parameter Ω_m and holographic dark energy density parameter Ω_Λ are given by

$$\Omega_m = \frac{\rho_m}{4H^2} = \frac{4k_2}{\alpha_1\beta^2 e^{\beta_1 t}}, \tag{71}$$

$$\Omega_\Lambda = \frac{\rho_\Lambda}{4H^2} = \frac{3\alpha}{4}. \tag{72}$$

From equations (71) and (72), we get overall density parameter

$$\Omega = \Omega_m + \Omega_\Lambda = \frac{4k_2}{\alpha_1\beta^2 e^{\beta_1 t}} + \frac{3\alpha}{4}. \tag{73}$$

7. Our findings:

i) Power Law Model:

At an initial epoch, both the scale factors vanish, start evolving with time and finally as $t \to \infty$ they diverge to infinity. This is consistent with the big bang model. As scale factors diverge to infinity at large time there will be Big rip at least as far in the future. In power law model, the scale factors vanish at $t = 0$ and hence the model has the initial singularity[71]. It is observed that the volume of the universe expands indefinitely for all positive values of b. The directional Hubble parameters are dynamical. These are diverse at $t = 0$ and approach zero monotonically at $t \to \infty$. The Hubble parameter decreases with time as shown in figure 1.We observe that the Hubble parameter, Expansion Scalar and Shear Scalar are very large at an initial epoch and finally tends to zero as $t \to \infty$ [72-75]. This suggested that at initial stage of the Universe, the expansion of the model is much faster and then slow down for later time this shows that the evolution of the Universe starts with infinite rate and with the expansion it declines. From the value of mean anisotropic parameter in Eq. (46), it is clear that the universe was anisotropic at early stage of evolution and approach to isotropy at large time as shown in figure 2. The rate of expansion of the universe decreases with time as shown in figure 3. The shear ratio of shear scalar to expansion scalar shows that at early epoch the universe is anisotropic and as time increases it tends to isotropy. The universe starts with an infinite rate of expansion and measure of anisotropy. This is consistent with big bang model. It is mentioned that q was supposed to be positive initially but recent observations from the supernova experiments suggest that it is negative. The positive deceleration parameter corresponds to a decelerating model while the negative value provides inflation. For $b > 4$ the deceleration parameter is negative which is specified in figure 4. The model (42) represents an accelerated universe. The physical parameters ρ_Λ, p_Λ are decreasing functions of time. They all become infinite at $t = 0$ and vanish for $t \to \infty$. The values of total energy density parameter $\Omega > 1$, $\Omega = 1$, $\Omega < 1$ correspond to the open, flat and closed universe respectively. From the right hand side of Eq. (56), one can observe that the overall density parameter approaches to approaches to constant quantity as $t \to \infty$. In power law expansion of the Universe, it is observed that the energy density of matter is always positive and decreasing function of time t. At the initial stage $t \to 0$ the Universe has infinitely large energy density $\rho_m \to \infty$ but with the expansion of the Universe it declines and at very large $t \to \infty$, it is null $\rho_m \to 0$.

ii) Exponential Law Model:

The scale factor are constant near $t = 0$, afterwards start increasing with time and as $t \to \infty$, they diverge to infinity. The model is free from singularity [76-78]. Hence in this case, the volume of the universe is an exponential function which expands with increase in time from a constant to infinitely large. The spatial volume is finite at $t = 0$. It expands exponentially as t increases and becomes infinitely large as $t \to \infty$. We have obtained the deceleration parameter $q = -1$ and $\frac{dH}{dt} = 0$ for this model. Hence, it provides the best values of the Hubble parameter and also the quickest rate of growth of the universe. The model may represent the inflationary era in the early universe and the very late time of the universe. The directional Hubble parameters are finite at $t = 0$ and $t = \infty$. The mean Hubble parameter is constant whereas the directional Hubble parameters are dynamical. The expansion scalar is constant throughout the evolution of the universe which exhibits uniform exponential expansion. The ratio of shear scalar to expansion scalar is non zero i.e. the universes is anisotropic and as time increases it tends to zero i.e. at late time the universe tending to isotropy. As t increases, the anisotropy of the expansion (Δ) decreases exponentially to null. Thus the space approaches to isotropy in this model. At $t = 0$, the anisotropy parameter is constant and decreases with time for $\beta_1 > 0$. It means that the universe was anisotropic at early stage and approaching to isotropy as time increases which is shown in figure 5.The Shear Scalar $\sigma \to 0$, as $t \to \infty$ as shown in figure 6.The sign of q indicate whether the universe accelerates or decelerates. A positive sign of q corresponds to the standard decelerating model and the negative sign of q indicate acceleration. Cosmological observations indicated that the expansion of the universe is accelerating at the present and it was decelerating in the past. From Eq. (66), it is observed that the deceleration parameter is negative i.e. the universe is accelerating which is in agreement with current observations of SNe Ia and CMB[79-81]. The physical behavior of holographic dark energy density and pressure are constant. As pressure is negative, it indicates that the derived model is accelerating. It is interesting to observe that the holographic dark energy EoS parameter ω_Λ in equation (69) behaves like cosmological constant, this is mathematically equivalent to cosmological constant Λ. From Eq. (70) it is conclude that at the initial stage of the Universe the energy density is approaches to constant value and with the expansion of the Universe it is decreases and at large expansion it is null i.e. $\rho_m \to 0$. Thus, our derived Universe is free from big rip. The sum of the energy density parameter approaches to the value near about $\frac{3\alpha}{4}$ as $t \to \infty$. So at late times the Universe becomes flat.

II. Conclusions:

- In this paper we have investigated the role of two fluid minimally coupled in the evolution of the holographic dark energy with matter in $f(R,T)$ gravity for the five dimensional Kaluza-Klein space-time.
- The exact solution of the field equations have been obtained by assuming two different volumetric expansion laws in a way to cover all possible expansion: namely exponential and power law expansion.
- It is observed that, in power law, the model has an initial singularity while in exponential model, it is free from any type of singularity.
- In both the model the value of deceleration parameter is negative which indicates that the expansion of the Universe is accelerating.
- In both the models the overall density parameter tends to one at late times i.e. the Universe becomes flat which is compatible with the observational results.
- The power law model initially stable but with expansion it is unstable.

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